A Surface Impedance Formula for Improved Computation of Eddy Current Loss in Nonlinear Magnetic Materials

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A new surface impedance formula for nonlinear magnetic materials is presented. It is shown to well predict the surface eddy current loss at all applied field amplitudes and for a wide range of commonly used materials. Compared to previously published formulas the accuracy is significantly improved across the intermediate transition range. Only two or three easily extracted parameters describing the BH curve are required, thus implying a minimum amount of pre-processing needed.

***Index Terms*—Magnetic materials, Nonlinear equations.**

I. Introduction

Accurate computation of eddy current loss in conducting magnetic materials is of great practical importance in many applications where high magnetic fields are present. Due to the very short skin depth, a surface impedance boundary condition (SIBC) description is often possible, thus reducing computational effort. However, a complicating factor is the highly nonlinear BH curve of many materials. In the pioneering works of Agarwal [1] and Lowther and Wyatt [2] analytical expressions for the loss, or equivalently the real part of the surface impedance, in a fully saturated material were derived. Ever since, numerous formulas have been suggested that for all values of the applied magnetic field can be used to represent the surface impedance in time-harmonic simulations.

For arbitrary values of the surface magnetic field the problem cannot be solved analytically. Hence, most of the suggested formulations, e.g. in [3]-[4], consist of a linear combination of the known low-field (linear) and high-field (Agarwal) analytical limits with field-dependent weight functions. It seems, however, that such formulas often tend to overestimate the loss in the intermediate transition range. Moreover, some of them [4] also require numerical pre-processing of the BH curve to find the weight functions.

A recently proposed method even skips the interpolation technique and directly introduces a numerically calculated nonlinear correction factor that multiplies the surface loss density found from a simulation where the material is assumed to be linear [5]. Moreover there exist software vendors that have developed proprietary nonlinear SIBC formulations [6]. Few details are, however, revealed about the equations being used.

In the present paper we describe an approximate formula, valid for the whole range of applied magnetic field and for a wide class of commonly used ferromagnetic materials. It is not constructed as a linear combination of the low- and high-field limits and therefore does not suffer from an overestimated loss density in the transition region. The formula is based on the fact that, except maybe for low fields, most BH curves can fairly well be described by the so-called Fröhlich formula [7]:

$$\frac{B(H)}{\mu_0} = \frac{(\mu_r - 1)H}{1 + H/H_{knee}} + H, \quad H_{knee} = \frac{B_{\max}}{(\mu_r - 1)\mu_0}. \tag{1}$$

μ_r is the low-field relative permeability and B_{max} is the saturation flux density.

II. Surface impedance and loss density

The surface impedance Z is defined as the ratio between the tangential electric and magnetic harmonic fields, E_s and H_s respectively, at the surface

$$Z = E_s / H_s \tag{2}$$

and the surface loss density (in W/m^2) can be computed as

$$P = \mathrm{Re}(Z) H_s^2 / 2. \tag{3}$$

The analytical solutions (assuming a sinusoidally varying H_s,) in the low-field (linear) and high-field (saturated) limits are, respectively, given by [1]

$$\mathrm{Re}(Z_l) = \sqrt{\frac{\omega\mu}{2\sigma}}, \quad \mathrm{Im}(Z_l) = \mathrm{Re}(Z_l), \tag{4}$$

$$\mathrm{Re}(Z_{Ag}) = \frac{16}{3\pi}\sqrt{\frac{\omega B_{\max}}{2\sigma H_s}}, \quad \mathrm{Im}(Z_{Ag}) = \frac{1}{2}\mathrm{Re}(Z_{Ag}), \tag{5}$$

In order to evaluate the accuracy of the different analytic surface loss formulas we also, for each material and for each value of H_s, carry out a truly nonlinear transient 1D simulation with a given tangential surface magnetic field $H_{tan} = H_s \sin(\omega t)$. The FEM software COMSOL Multiphysics® [8] is employed for this task. The resulting time average of the total loss is the reference value to which the analytical formulas are compared in the present work.

III. New loss formula for Fröhlich BH curves

The proposed loss expression, normalized with the low-field value (4) using the relative permeability μ_r, is for a Fröhlich curve given by

$$\frac{\mathrm{Re}(Z)}{\mathrm{Re}(Z_l)} = \frac{1}{\left[1 + \left(\frac{\mathrm{Re}(Z_l)}{\mathrm{Re}(Z_{Ag})}\right)^{1.6}\right]^{(1/1.6)}}. \tag{6}$$

Note that (6) automatically approaches (4) and (5) for low and high fields, respectively.

In Fig. 1 the exact, transient solution is compared to the new SIBC (6) as well as to the formulas suggested in [3] and [4].

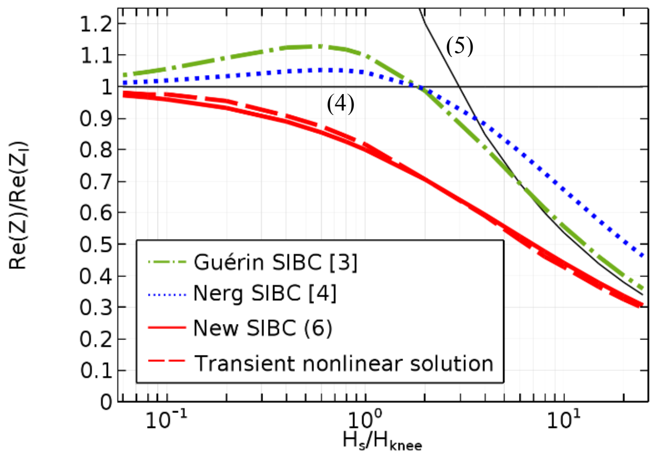

Fig. 1. Normalized real part of surface impedance as function of normalized applied surface magnetic field. Calculated for Fröhlich BH curves (1). The two limiting expressions (4) and (5) are also shown normalized.

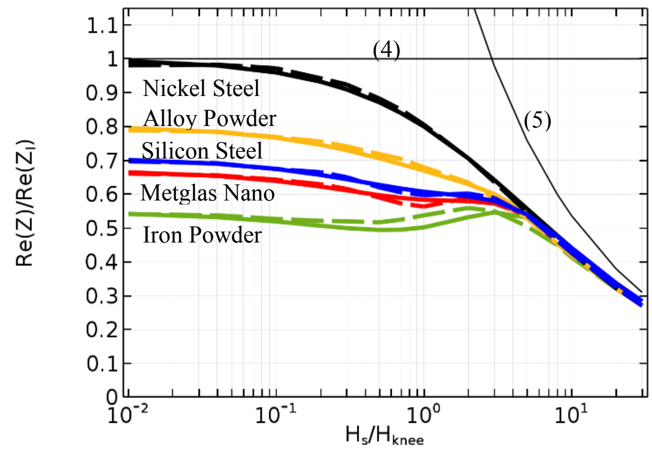

Fig. 3. Normalized real part of surface impedance for the five BH curves shown in Fig. 2. Dashed curves denote transient nonlinear solution and solid curves the loss according to (7).

IV. New Loss Formula for General BH Curves

From domain theory [7] and experiments it is known that the Fröhlich expression (1) does not represent the behavior at very low field strengths, where the local permeability μ_{r0} is usually lower than the parameter μ_r occurring in (1). To take this into account we propose an improved version of (6):

$$\frac{\mathrm{Re}(Z)}{\mathrm{Re}(Z_l)} = \frac{1}{\left[1+\left(\frac{\mathrm{Re}(Z_l)}{\mathrm{Re}(Z_{Ag})}\right)^{1.6}\right]^{(1/1.6)}} - \left(1-\sqrt{\mu_{r0}/\mu_r}\right)\exp\left[-(H_s/(1.73\cdot H_{knee}))^{1.2}\right], \qquad (7)$$

where Z_l is calculated using μ_r. The accuracy of (7) is now evaluated by comparing with transient nonlinear simulations of typical magnetic materials. To do this we pick five representative measured BH curves, all very different in terms of B_{max}, μ_r, and μ_{r0}, from the material library of COMSOL Multiphysics® and fit these curves to a generalization of (1), allowing for $\mu_{r0} < \mu_r$. The fitted normalized curves are plotted in Fig. 2 and the corresponding calculated loss curves are shown in Fig. 3, where dashed and solid curves represent transient solutions and (7), respectively.

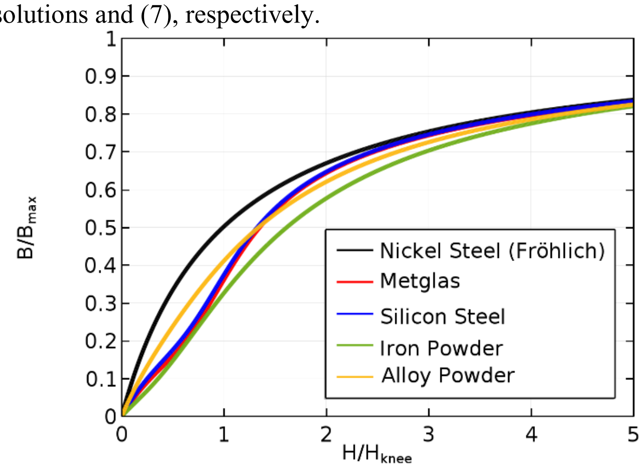

Fig. 2. Fitted BH curves of five real world materials.

V. Conclusion

A number of nonlinear SIBC:s have been presented over the years but surprisingly little data have been published regarding their accuracy. Here, we have demonstrated that the formula (7) yields loss results agreeing, within 5% or less, with more detailed transient simulations. The necessary BH curve parameters B_{max}, μ_r, and μ_{r0} can be determined either by direct inspection or by applying numerical fitting.

So far, only the commonly studied case of a sinusoidally varying surface magnetic field has been considered. The more general case with significant harmonics needs further investigation. However, the fact that the high field limit losses for sinusoidal magnetic and electric surface fields differ by only 5% [2]-[3] indicates that (6) and (7) may work also for rather general excitations.

Finally, we note that (6) and (7) can also be used to give Im(Z) in an analogous manner. The full complex surface impedance is therefore known and can be utilized as a boundary condition in time-harmonic simulations.

VI. References

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